HEAT TRANSFER IN A SYSTEM OF PARALLEL AXISYMMETRIC RIBS UNDER CONDITIONS OF A CHANGE IN THE AGGREGATE COOLANT STATE

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An analytical method is developed for analyzing the nonstationary processes in a heat exchanger. Computational relationships are obtained and criteria are formulated for the thermal optimization of the system parameters.

Heat exchange structures in the form of axisymmetric ribs fastened to a central cooler and submerged in a melting, subliming, or freezing coolant [1-3] are utilized extensively in a number of modern branches of engineering, cryogenic, space, radio engineering, etc., for the thermostatting of different objects. The problem of optimizing the structural, thermal, and mass-scale parameters of such apparatus is made difficult by the lack of analytical solutions describing the nonstationary thermal fields and also by the sufficient complexity of using a numerical solution [2].

The purpose of this paper is to obtain an analytic solution of the problem under consideration in application to sublimation cold accumulators and melting-solidifying accumulators [1-3] when a constant power is delivered (removed) to a heat exchange system (refrigerator, electrical heater, external influx to optical or other cryogenic units).

For definiteness we examine the temperature fields of a system of parallel circular ribs of radius R_2 and thickness δ (see Fig. 1) fastened to a central cooler of radius R_1. The space of width 2h ($\delta \ll h \ll R_2$) between the ribs is filled with a subliming coolant. A constant thermal flux Q is delivered to the central cooler. We consider the thermal resistance of the cooler to be much less than the thermal resistance of the ribs R_r. Consequently, the heat supply to each rib Q_{R_1} can be constant in time, and just the problem of sublimation in the gap between adjacent ribs (see Fig. 1) can be considered independently. Then the temperature field of the disc T and the width of the gas gap H(R, t) along it are described in a cylindrical coordinate system (z is along the disc axis, and R along the slot radius) in conformity with the approaches used in [1, 2] by the following system of equations

$$c_M\rho_M \frac{\partial T}{\partial t} = \lambda_M \frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial T}{\partial R}\right) - \frac{2}{\delta} q_z(R, t); \qquad (1)$$

$$T(R, 0) = T_0; \qquad (2)$$

$$q_z(R, t) = \lambda_e \frac{T(R, t) - T_0}{H(R, t)} = \rho_0 r_s \frac{\partial H}{\partial t}(R, t); \qquad (3)$$

$$q_R(R_1, t) = -\lambda_M \frac{\partial T}{\partial R}(R_1, t) = q_0 = \frac{Q_{R_1}}{2\pi R_1\delta}; \qquad (4)$$

$$q_R(R_2, t) = -\lambda_M \frac{\partial T}{\partial R}(R_2, t) = 0. \qquad (5)$$

Following [1, 2], the following assumptions are made for writing (1)-(5). The heat conduction λ_M and the specific heat c_M of the plate are constant while the temperature field is homogeneous along the plate thickness δ ($\delta \ll R_2 - R_1$) and depends only on the coordinate R and the time t. The heat flux $q_z(R, t)$ from the plate surface is expended completely on surface sublimation of the coolant (with density ρ_0 and specific heat of

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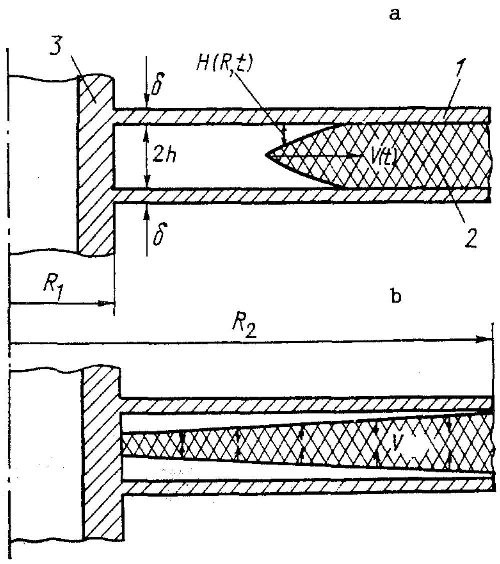

Fig. 1. Configurations of the main element of a sublimation cold accumulator structure with axisymmetric ribs: 1) rib; 2) coolant; 3) heat pipeline, phase transition front is displaced; a) in the radial direction for $R_r \ll 1$; and b) perpendicularly to the plane of the plate for $\bar{R}_r >> 1$.

sublimation r_s) and is determined by the effective heat conduction of the gas gap λ_e. The rib temperature is constant at the initial instant and equals the temperature T_0 of the subliming body. We shall also neglect heat exchange with the central cooler in conformity with the numerical data [2] and the duration of the sublimation process around it.

Moreover, taking into account the slow change in the aggregate state of the medium, the process of a change in the gas gap profile and the plate temperature will be assumed quasistationary. There results from a further solution of the problem that within the framework of the assumptions made, the nature of the process mentioned depends in principle on the value of the criterion

$$\bar{R}_r = (\lambda_M \delta h/[\lambda_e \, (R_2^2 - R_1^2) \ln (R_2/R_1)])^{1/2}, \tag{6}$$

which physically governs the relationship between the thermal resistances of the gas gap between the plates R_g and the plate itself R_p in the radial direction

$$R_g = h/[2\pi\lambda_e \, (R_2^2 - R_1^2)]; \tag{6a}$$

$$R_p = \ln (R_2/R_1)/(2\pi\delta\lambda_M). \tag{6b}$$

A gas gap is formed rapidly in the domain $\bar{R}_r \gg 1$ above the whole plate surface and then it starts to grow with time at a constant velocity (see Fig. 1b). The other limit case is observed in the domain $\bar{R}_r \ll 1$ when a sublimation front advancing along the rib radius (see Fig. 1a) is formed from the side of the central cooler. Such an extraordinary pattern will be observed physically when the thermal resistance of the gas gap between the ribs is much less than the thermal resistance of the rib itself, i.e., when quite thin ribs are arranged close to each other on the cooler. It is interesting to note that realization of the "mean-distributed" or "local" phase transition regimes under consideration is independent of the intensity of the heat supply and development of the sublimation process after formation of a quasistationary front in both the one and the other case. The pattern described should evidently be observed qualitatively even in heat exchange apparatus of other symmetry, for instance, between global surfaces or in a rectangular rib system.

Let us turn to the solution of the "local" sublimation problem governed by values of the parameter $\bar{R}_r \ll 1$. In this case the working substance sublimation process ceases in the domain $R \le R_1(t)$ (see Fig. 1a) where the thickness of the gas gap at the rib reaches the value h, and if heating of the coolant gas phase is neglected, it is necessary to require

$$q_z(R,\ t)|_{R \leqslant R_1(t)} = 0.$$

An adiabatic approach can be applied to describe the regime when the sublimation front has already been formed and its subsequent displacement is determined mainly by heat flux from the rib surface by examining the interval between t and t + τ, where t is of one order and τ is small compared with the characteristic times of phase transition front motion, i.e., $\tau \ll R_1(t)(dR_1(t)/dt)^{-1}$.

Let us seek the approximate solution $T(R, R_1(t), \tau)$ of the heat conduction equation (1) in the ring $R_1(t) \leq R \leq R_2(t)$ by limiting ourselves in the time τ to the first term in the expansion of the flux $q_z(t + \tau)$ in a series in the small parameter $\tau/R_1(t)(dR_1(t)/dt)$, i.e., by considering that q_z is independent of τ

$$q_z(R,\ R_1(t),\ \tau) \simeq q_z(R,\ R_1(t)). \tag{7}$$

We then have from (3)

$$H(R,\ R_1(t),\ \tau) \simeq H_0(R,\ R_1(t)) + \tau q_z(R,\ R_1(t))/(\rho_0 r_s) \tag{8}$$

and

$$T(R,\ R_1(t),\ \tau) \simeq T_0 + \\ + \frac{1}{\lambda_e} q_z(R,\ R_1(t))\,[H_0(R,\ R_1(t)) + \tau q_z(R,\ R_1(t))/(\rho_0 r_s)]. \tag{9}$$

Substituting (9) into (1) and neglecting components containing the small quantity $dR_1(t)dt$, we obtain a system of equations for the functions $q_z(R)$ and $H_0(R)$ that must be solved for "adiabatic" boundary conditions

$$\left.\frac{\partial T}{\partial R}\right|_{R=R_2(t)} = 0,\ H_0(R,\ R_1(t))|_{R=R_2(t)} = 0,\ H_0(R,\ R_1(t))|_{R=R_1(t)} = h, \tag{10}$$

supplemented by continuity conditions for the temperature $T(R, R_1(t), \tau)$ and the heat flux $-\lambda_M(\partial T/\partial R)(R, R_1(t), \tau)$ in the circle $R = R_1(t)$.

The heat expenditure in sublimation is zero in the domain $R_1 \leq R \leq R_1(t)$ and the solution of (1) is sought for $q_z = 0$ in the form

$$T(R,\ R_1(t),\ \tau) = T(R,\ R_1(t),\ 0) + \frac{\partial T}{\partial \tau}(R,\ R_1(t),\ 0)\,\tau, \tag{11}$$

that satisfies the boundary condition (4).

As a result of integrating the heat conduction Eq. (1) we arrive, firstly, at the deduction that the heat flux from the rib is distributed uniformly in the domain $R_1(t) = R = R_2(t)$ with intensity $C_0(R_1(t))$, i.e.,

$$q_z(R,\ R_1(t)) \equiv C_0(R_1\ (t)). \tag{12}$$

Secondly, we find the dependence of the magnitude of the gas gap $H_0(R, R_1(t))$ on the radius R

$$H_0(R,\ R_1(t)) = \frac{1}{4} A R_2^2(t) \left(\frac{R^2}{R_2^2(t)} - 1 - \ln \frac{R^2}{R_2^2(t)}\right), \tag{13}$$

where

$$A = \frac{1}{\lambda_M}\left(\frac{c_M \rho_M}{\rho_0 r_s} C_0(R_1(t)) + \frac{2}{\delta}\lambda_e\right), \tag{14}$$

and the outer coordinate of the front $R_2(t)$ is determined by the inner $R_1(t)$ and the heat flux intensity $C_0(R_1(t))$ according to the equation

$$\frac{R_1^2(t)}{R_2^2(t)} - 2\ln\frac{R_1(t)}{R_2(t)} = 1 + \frac{4h}{A R_2^2(t)}. \tag{15}$$

In the domain $R_1 = R = R_1(t)$ the temperature distribution corresponding to the conditions listed above has the form

$$T(R,\ R_1(t),\ \tau) = T_0 + \frac{h}{\lambda_e} C_0(R_1(t)) + \tau \frac{C_0^2(R_1(t))}{\lambda_e \rho_0 r_s} - \frac{R_1}{\lambda_M} \times$$

$$\times\left(q_0+\frac{c_M\rho_M R_1}{2\lambda_e\rho_0 r_s}C_0^2(R_1(t))\right)\ln\frac{R}{R_1(t)}+\frac{c_M\rho_M C_0^2(R_1(t))}{4\lambda_M\lambda_e\rho_0 r_s}(R^2-R_1^2(t)). \tag{16}$$

Continuity of the radial heat flux for $R = R_1(t)$ is assured by satisfying the condition

$$\frac{c_M\rho_M\delta C_0(R_1(t))}{2\rho_0 r_s\lambda_e}(R_2^2(t)-R_1^2)-\frac{q_0R_1\delta}{C_0(R_1(t))}=R_1^2(t)-R_2^2(t), \tag{17}$$

which, together with (15), yields the dependence of the heat flux density $C_0(R_1(t))$ in the ring $R_1(t) \le R \le R_2(t)$ on the coordinate $R_1(t)$ of the front "tail".

There now remains for us to find an explicit dependence of $R_1(t)$ on the time t, and the problem of seeking the temperature distribution in a plate ((9), (12), (13), (16)) will be solved. To do this we turn to (8) and rewrite it in the form

$$\tau C_0(R_1(t))/(\rho_0 r_s)=H_0(R,\ R_1(t)+\Delta R_1(t))-H_0(R,\ R_1(t)),$$
$$\Delta R_1(t)=R_1(t+\tau)-R_1(t).$$

Noting that the total quantity of heat having departed to sublimation in the small time τ equals

$$C_0(R_1(t))\pi\tau(R_2^2(t)-R_1^2(t))=2\pi\rho_0 r_s\Delta R_1(t)\int\limits_{R_1(t)}^{R_2(t)}dRR\frac{\partial H_0(R,\ R_1(t))}{\partial R_1(t)},$$

and letting τ tend to zero, we obtain a differential equation for $R_1(t)$

$$\frac{dR_1(t)}{dt}=\frac{C_0(R_1(t))}{2\rho_0 r_s}(R_2^2(t)-R_1^2(t))\left[\int\limits_{R_1(t)}^{R_2(t)}dRR\frac{\partial H_0(R,\ R_1(t))}{\partial R_1(t)}\right]^{-1}. \tag{18}$$

The solution obtained is simplified substantially if it is taken into account that specific heat c_M of the plate is so small

$$c_M\ll\frac{4\rho_0 r_s(\lambda_M\lambda_e h)^{1/2}}{\rho_M q_0R_1^2\delta^{3/2}},$$

that it can be neglected in (15)-(17). Then (15), (17), and (18) are rewritten in the form

$$R_1^2(t)-R_2^2(t)-2R_2^2(t)\ln\frac{R_1(t)}{R_2(t)}=2h\delta\frac{\lambda_M}{\lambda_e}, \tag{19}$$

$$C_0(R_1(t))=\frac{q_0R_1\delta}{R_2^2(t)-R_1^2(t)}, \tag{20}$$

$$\frac{dR_2(t)}{dt}=\frac{q_0R_1\delta^2\lambda_M}{\rho_0 r_s\lambda_e}\frac{1}{R_2(t)}\left(R_2^2(t)-R_1^2(t)+2R_1^2(t)\ln\frac{R_1(t)}{R_2(t)}\right)^{-1}. \tag{21}$$

For values of $\bar{R}_r \ll 1$ the solution of (19) yields the magnitude of the "local" sublimation domain

$$R_2(t)-R_1(t)\simeq\sqrt{\gamma},\ \gamma=\frac{\lambda_M}{\lambda_e}h\delta. \tag{22}$$

The relationship (22) is satisfied at almost all times of thermoaccumulator operation if $\sqrt{\gamma} \ll R_2 - R_1$. The dynamics of the inner and outer boundaries of the phase transition domain is here described by equations of one kind

$$R_i(t)\frac{dR_i(t)}{dt}=V_0R_1,\ \ i=1,\ 2, \tag{23}$$

where $V_0 = q_0\delta/(2\rho_0 r_s h)$.

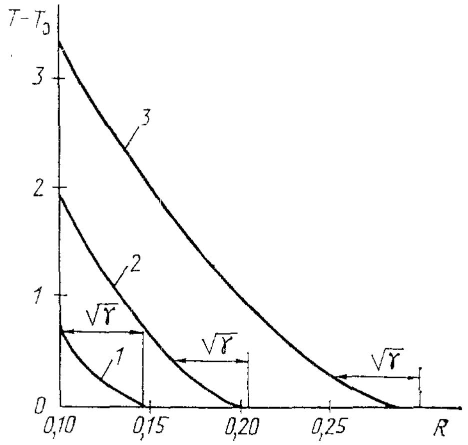

Fig. 2. Temperature distribution over the rib of an argon accumulator with "local" sublimation section $\sqrt{\gamma} \simeq 0.048$ m ($\bar{R}_r^2 = 0.24$; $Q_{R_1}{}^1 = 0.05$ W) at different times t: 1) at the time of front formation, $t_0 = 6.5$ days; 2) t = 43 days; 3) t = 132 days. $T - T_0$, K; R, m.

Integrating (23) for i = 1 we obtain an explicit dependence of $R_1(t)$ on the time t

$$R_1(t) = R_1(t_0)\left[1 + \frac{2R_1V_0}{R_1^2(t_0)}(t - t_0)\right]^{1/2}, \quad \sqrt{\gamma} \ll R_2 - R_1. \tag{24}$$

The radial temperature distribution in the plate is represented in Fig. 2 for different times t with (24) taken into account.

The most important characteristic of a sublimation accumulator is the temperature stability of the object being cooled during exploitation. In the case $\bar{R}_r \ll 1$ the temperature of the object (heat line) is determined by the sum of the temperature drops over the rib ΔT_r (in the domain $R_1 \le R \le R_1(t)$) and in the "local" sublimation section of the front ΔT_f (in the domain $R_1(t) \le R \le R_2(t)$).

The temperature drop over the rib grows logarithmically with time as a function of $R_1(t)$

$$\Delta T_r(t) = \frac{q_0 R_1}{\lambda_M} \ln \frac{R_1(t)}{R_1}, \tag{25}$$

while the temperature drop over the length of the sublimation front diminishes with time in inverse proportion to the radius of the domain in which the sublimation of the working substance ceased

$$\Delta T_f(t) \simeq \frac{R_1 q_0 \sqrt{\gamma}}{\lambda_M (R_1(t) + R_2(t))}. \tag{26}$$

It is easy to see that $\Delta T_f \ll \Delta T_r$ for almost all values of the time t (see Fig. 3).

For the particular case $\sqrt{\gamma} \ll R_2 - R_1$ considered here, it is possible to estimate the contribution to the magnitudes being determined for the higher order components from the expansion of $q_z(t + \tau)$ in powers of τ [see (7)]. Thus, for instance, taking account of the quantity $(dq_z/d\tau)_{t=0}\tau$ in (7) results in corrections on the order of $\delta R_1/R_1^2(t)$ in the expressions (24)-(26) that guarantee the validity of these expressions for $\delta R_1 \ll R_1^2(t)$.

In the opposite limit case $\bar{R}_r \gg 1$, as has already been noted, rapid formation of a gas gap whose thickness varies along the radius R according to the law

$$H_{av}(R) = \frac{\lambda_e}{\delta\lambda_M}\left(\frac{R^2 - R_2^2}{2} - R_2^2 \ln \frac{R}{R_2}\right) \tag{27}$$

occurs over the whole plate surface.

Starting with the time $\tau = 0$ of the appearance of a slot between the solid phase and the outer endface of the plate, the gas gap increases with an identical velocity $C_{av} = q_0\delta R_1/(\rho_0 r_s(R_2^2 - R_1^2))$ for all values of the radius R. A constant and uniformly distributed flux $q_z = q_0\delta R_1/(R_2^2 - R_1^2)$ over the plate area for which the plate temperature field is described by the simple formula

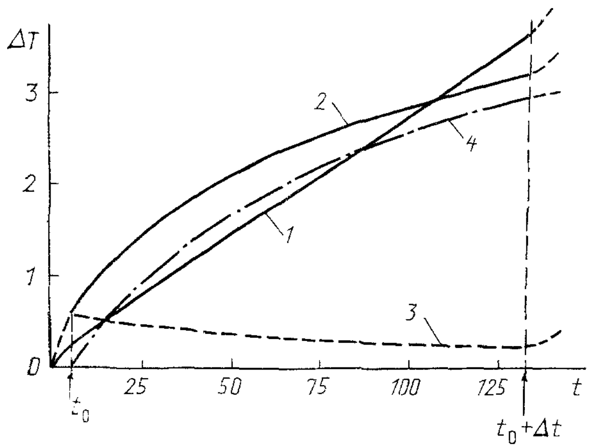

Fig. 3. Dependence of the temperature drop (between the solid argon and the heat line) and its components on the time: 1) in the case of "mean-distributed" sublimation ($\bar{R}_r{}^1 = 7.11$; $Q_{R_1}{}^1 = 0.37$ W); 2) $\Delta T_\Sigma = \Delta T_f + \Delta T_r$ in the case of "local" sublimation ($\bar{R}_r{}^2 = 0.24$; $Q_{R_1}{}^2 = 0.05$ W); 3) ΔT_f on the "local" sublimation section; 4) ΔT_r, ΔT, K; t, days.

$$T(R,\ \tau) = T_0 + \frac{q_0\delta R_1}{(R_2^2 - R_1^2)\lambda_e}\left[H_{av}(R) + \frac{q_0\delta R_1\tau}{\rho_0 r_s\,(R_2^2 - R_1^2)}\right] \tag{28}$$

corresponds to such motion.

Therefore, by variating the ribbing parameters δ, h, and λ_M any of the regimes considered $\bar{R}_r \gg 1$ or $\bar{R}_r \ll 1$ can be assured. By using the known relationship

$$Q_{ri} = \Delta T/R, \tag{29}$$

the thermal resistance of the gas gap between the ribs R_g and of the very rib in the radial direction that are described by the relationships (6a) and (6b) can be determined for these regimes from (25) and (28). Just one of the resistances R_g or R_p is evidently conserved in these regimes precisely because of their noncommensurability determined by the conditions $\bar{R}_r \gg 1$ or $\bar{R}_r \ll 1$. Both these resistances must be taken into account in the intermediate domain $\bar{R}_r \sim 1$ encountered most often in practice, and it can be assumed approximately that the total resistance R_Σ equals

$$R_\Sigma = R_g + R_p, \tag{30}$$

while the total temperature drop between the coolant and the heat line can be determined in the domain $10 > \bar{R}_p > 0.1$ in the form

$$\Delta T_\Sigma = Q_{ri} R_\Sigma. \tag{31}$$

It hence follows that the minimal temperature drops in a heat exchanger can be achieved only under conditions of a limit diminution of the total thermal resistance R_Σ and also of both components R_g and R_p, respectively. This means that a heat exchanger with $R_p \approx R_g$ must be constructed to obtain the minimal drop ΔT.

The above elucidation permits proposing the following algorithm for selection of the optimal construction parameters. We select the width of the gap between the plates equal to

$$h \approx \pi\lambda_e\,(R_2^2 - R_1^2)\,\frac{\Delta T_\Sigma}{Q_{ri}}\ . \tag{32}$$

from relationships (6a), (30), and (31).

Furthermore, we select the rib thickness

$$\delta \approx \frac{\ln (R_2/R_1)}{\pi\lambda_M} \frac{Q_{ri}}{\Delta T_\Sigma} \quad (33)$$

from the relationship (6b).

Therefore, the condition

$$h\delta = \frac{\lambda_e}{\lambda_M} (R_2^2 - R_1^2) \ln (R_2/R_1) \quad (34)$$

is satisfied for a heat exchanger that is optimal in the temperature drop, which is equivlent to $\overline{R}_p = 1$. At the same time, the ratio h/δ is not kept constant but is a quadratic function of the ratio $\Delta T_\Sigma/Q_{ri}$:

$$\frac{h}{\delta} \approx \frac{\lambda_M\lambda_e \pi^2 (R_2^2 - R_1^2)}{\ln (R_2/R_1)} \left(\frac{\Delta T_\Sigma}{Q_{ri}}\right)^2. \quad (35)$$

The second important characteristic of the system is the residual mass Δm (per unit length of the cooler) at the time when (9) and (28) cease to describe the temperature fields (see Fig. 3). We approximately obtain that for the two systems lying in the range $\overline{R}_r{}^1 \gg 1$ and $\overline{R}_r{}^2 \ll 1$, the mass relationship equals

$$\Delta m^1/\Delta m^2 \approx B/[\overline{R}_r^2 (\overline{R}_r^1)^2],\ B \sim 1. \quad (36)$$

As an illustration, we take a system of copper ribs with $\overline{R}_r{}^1 \simeq 7.11$ ($\lambda_M{}^1 = 5\cdot 10^2$ W/(m·K), $R_2 = 0.3$ m, $R_1 = 0.1$ m, $\delta_1 = 8\cdot 10^{-4}$ m, $h_1 = 2.7\cdot 10^{-2}$ m) placed in subliming argon. Replacing the rib material by the stainless steel 12Kh18N9T [$\lambda_M{}^2 = 8.2$ Wt/(m·K)] and altering the parameters δ and h somewhat ($\delta_2 = 3\cdot 10^{-4}$ m, $h_2 = 5\cdot 10^{-3}$ m), we obtain $\overline{R}_r{}^2 \simeq 0.24$. In sum we have the relationship $\Delta T_1 \simeq 1.13\ \Delta T_2$, while $\Delta m^1 \simeq 0.05\ \Delta m^2$. Therefore, in the case of nonrigid requirements on the temperature stability, it is possible to go over to the range $\overline{R}_r \gg 1$ to reduce the residual masses.

In conclusion, the following fundamental results can be noted. Analytic relations are obtained in a quasistationary approximation to describe changes in the temperature field and the thermal resistances of a system of circular ribs submerged in a medium undergoing a phase transition. An algorithm is proposed for the selection of the structural parameters of the system. It is shown that the condition for thermal optimization of the system is selection of the rib and gas gap thicknesses that will permit commensurability of the rib and gas gap thermal resistances for a given temperature drop and thermal load.

NOTATION

T(R, t) is the rib (fin) temperature field; H(R, t) is the running value of the height of the gas gap between the rib and the interfacial surface of different coolant phases; T_0 is the phase transition temperature; q_0 is the heat flux density at the rib base; r_s is the specific heat of sublimation (melting) of the working substance; λ_M is rib heat conduction; c_M is the rib specific heat; ρ_M is the density of the rib material; ρ_0 is the coolant density; δ is the rib thickness; h is half the spacing between ribs; R_1 and R_2 are the rib inner and outer radii; $R_1(t)$ and $R_2(t)$ are coordinates of the inner and outer boundaries of the "local" sublimation zone, and λ_e is the effective heat conduction of the coolant gas phase.

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